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Department of Computer Science

Question Bank, 2023-2024

Subject: Mathematics (Linear Algebra)

Class: B.Sc. CS. (Entire) - II

- 1) Show that intersection of two subspaces is again a subspace.
- 2) State and prove triangle inequality.
- 3) A non-empty subset S of a vector space V over the field F is subspace of v iff $\alpha x + \beta y \in S$, for all $\alpha, \beta \in F$ & $x, y \in S$
- 4) Solve the following system using Gauss- elimination method.

$$x_1 + x_2 + x_3 = 4$$

$$x_1 - x_2 + 2x_3 = 3$$

$$2x_1 + 3x_2 - x_3 = 6$$

- 5) Verify Cayley – Hamilton theorem for the matrix.

$$A = \begin{bmatrix} 3 & -2 & -1 & 2 \end{bmatrix}$$

- 6) Show that $S = \{(1,0,0), (0,1,0), (0,0,1)\}$ is a basis of R^3
- 7) State & prove Cauchy Schwarz's inequality.
- 8) Find eigen values and eigen vectors for the following matrix.

$$A = \begin{bmatrix} 2 & 1 & 0 & 0 & 1 & -1 & 0 & 2 & 4 \end{bmatrix}$$

- 9) Let $S = \{v_1, v_2, \dots, v_n\}$ be a set of vectors in a vector space V , then prove that
 - (I) $L(S)$ is a subspace of v .
 - (II) $L(S)$ is the smallest subspace of v containing S .

10) If $T : V \rightarrow U$ is a linear transformation then show that range of T is a subspace of U .

11) State and prove triangle inequality.

- 12) A non-empty subset S of a vector space V over the field F is a subspace of V iff $cx + \beta y \in S$, for all $c \in F$ & $x, y \in S$
- 13) Show that the intersection of two subspaces is again a subspace.
- 14) Obtain A^{-1} , if it exists for

$$A = \begin{bmatrix} 3 & 1 & 5 & 2 & 4 & 1 \\ - & 4 & 2 & - & 9 \end{bmatrix}$$
- 15) Let A be an $n \times n$ matrix. The following are equivalent
 (a) A is invertible
 (b) $AX=B$ is consistent, for every $n \times 1$ matrix B
- 16) Find the value of α for which the system

$$\begin{aligned} \alpha x + y + z &= 1 \\ x + \alpha y + z &= 1 \\ x + y + \alpha z &= 1 \end{aligned}$$
- 17) Prove that Every orthogonal set of non zero vectors in an inner product Space is linearly independent.
- 18) Define: (I) Basis
 (II) Dimension
- 19) Consider the Euclidean inner product space R^2 . Transform the basis $\{u_1, u_2\}$ into an orthonormal basis, where

$$u_1 = (4, -3), \quad u_2 = (1, -1)$$
- 20) Define : (I) Kernel of T
 (II) Range of T
- 21) Let $T: V \rightarrow W$ be a linear transformation, then prove that
 $T(-u) = -T(u)$, for every $u \in V$
- 22) If $T: V \rightarrow W$ is a linear transformation, then prove that $\ker(T)$ is a subspace of V
- 23) If v_1, v_2, \dots, v_n are vectors in a vector space V , then prove that the set W of all Linear combinations of v_1, v_2, \dots, v_n is a subspace of V
- 24) Define linear span. Give one example.

- 25) If $T: V \rightarrow W$ is a linear transformation, then prove that $R(T)$ is a subspace of V
- 26) If u & v are any two vectors in an inner product space, then

$$\langle u, v \rangle^2 \leq \langle u, u \rangle \langle v, v \rangle$$
- 27) State & prove triangle inequality.
- 28) Determine whether the following system has unique solution or not

$$\begin{aligned} p+3q+5r+s &= 0 \\ 4p-7q-3r-s &= 0 \\ 3p+2q+7r+8s &= 0 \end{aligned}$$
- 29) Prove that : A system of linear equation has no solution, exactly one solution Or infinitely many solution.
- 30) Define : (i) system of linear equations
(ii) constant system
- 31) Reduce the following matrix to row echelon form

$$A = [1 \ 2 \ 3 \ 2 \ 5 \ 3 \ 1 \ 0 \ 8]$$
- 32) Define : (i) Eigen values
(ii) Eigen vectors
- 33) State & prove Pythagoras theorem
- 34) Define : (i) Linearly Independent
(ii) Linearly dependent
- 35) State & prove Cauchy – Schwarz inequality.
- 36) Find eigen values and eigen vectors for the following matrix.

$$A = [3 \ 1 \ 3 \ 2 \ 4 \ 2 \ 1 \ 1 \ 3]$$
- 37) Verify Cayley- Hamilton theorem for the following matrix.

$$[3 \ -7 \ 4 \ -5]$$
- 38) Define : (i) Vector Space
(ii) Linear transformation
- 39) State and prove triangle inequality.
- 40) Show that range of T Is a subspace of V